

## Vacuum Stability of Standard Model<sup>++</sup>

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### Abstract

The latest results of the ATLAS and CMS experiments point to a preferred narrow Higgs mass range ( $m_h \simeq 124 - 126$  GeV) in which the effective potential of the Standard Model (SM) develops a vacuum instability at a scale  $10^9 - 10^{11}$  GeV, with the precise scale depending on the precise value of the top quark mass and the strong coupling constant. Motivated by this experimental situation, we present here a detailed investigation about the stability of the SM<sup>++</sup> vacuum, which is characterized by a simple extension of the SM obtained by adding to the scalar sector a complex  $SU(2)$  singlet that has the quantum numbers of the right-handed neutrino,  $H''$ , and to the gauge sector an  $U(1)$  that is broken by the vacuum expectation value of  $H''$ . We derive the complete set of renormalization group equations at one loop. We then pursue a numerical study of the system to determine the triviality and vacuum stability bounds, using a scan of  $10^8$  random set of points to fix the initial conditions. We show that, if there is no mixing in the scalar sector, the top Yukawa coupling drives the quartic Higgs coupling to negative values in the ultraviolet and, as for the SM, the effective potential develops an instability below the Planck scale. However, for a mixing angle  $0.05 \lesssim \alpha \lesssim 0.35$ , with the new scalar mass in the range  $500 \text{ GeV} \lesssim m_{h''} \lesssim 5 \text{ TeV}$ , the SM<sup>++</sup> ground state can be absolutely stable up to the Planck scale. These results are largely independent of TeV-scale free parameters in the model: the mass of the non-anomalous  $U(1)$  gauge boson and its branching fractions.

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## I. INTRODUCTION

The CERN Large Hadron Collider (LHC) has begun a bound and determined exploration of the electroweak scale. Recently, the ATLAS [1] and CMS [2] collaborations presented an update of the Higgs searches, independently combining about  $5 \text{ fb}^{-1}$  of data collected at  $\sqrt{s} = 7 \text{ TeV}$  and more than  $5 \text{ fb}^{-1}$  at  $\sqrt{s} = 8 \text{ TeV}$ . The excess at 125 GeV that was evident already in data from the 7 TeV run [3, 4] has been consistently observed by both experiments in the  $\gamma\gamma$  invariant mass spectrum with a local significance of  $4.5\sigma$  and  $4.1\sigma$ , respectively. In addition, an excess of 4 leptons events (with  $m_{4\ell} \simeq 125 \text{ GeV}$ ) which can be interpreted as a signal of the  $h \rightarrow ZZ^* \rightarrow 4\ell$  decay, is observed by both experiments with a significance of  $3.4\sigma$  and  $3.2\sigma$ , respectively. The CMS experiment also presented updated Higgs boson searches in  $W^+W^-$  (a broad excess in the invariant mass distribution of  $1.5\sigma$  is observed),  $b\bar{b}$  (no excess is observed), and  $\tau\bar{\tau}$  (no excess is observed) channels. More recently, the ATLAS Collaboration reported a  $2.8\sigma$  deviation in the  $h \rightarrow W^+W^- \rightarrow 2\ell\nu$  decay channel [5]. When combining the data from the 7 TeV and 8 TeV runs, both experiments separately have reached the sensitivity to the new boson with a local significance of  $5\sigma$  [6, 7]. Very recently, the CDF and D0 collaborations published an update on searches for the Higgs boson decaying into  $b\bar{b}$  pairs, using  $9.7 \text{ fb}^{-1}$  of data collected at  $\sqrt{s} = 1.96 \text{ TeV}$  [8]. They reported a  $3.3\sigma$  deviation with respect to the background-only hypothesis in the mass range between 120 – 135 GeV.

All in all, LHC data strongly suggest that the observed state feed the electroweak symmetry breaking, and is likely the Higgs boson. However, it remains to be seen whether its trademarks, particularly the production cross section and decay branching fractions, agree with the very precise prediction of the Standard Model (SM). The decay channels that are most sensitive to new physics are the loop induced decays  $h \rightarrow \gamma\gamma$  and  $h \rightarrow \gamma Z$ . Interestingly, the most recent analyses [9, 10] of the combined LHC data seem to indicate that there is a deviation from SM expectations in the diphoton channel at the  $2.0\sigma - 2.3\sigma$  level; see, however, [11].

From a theoretical perspective some modification of the Higgs sector has long been expected, since the major motivation for physics beyond the SM is aimed at resolving the hierarchy problem. Even if one abandons such a motivation for new physics there are still enduring concerns about the stability of the electroweak vacuum, which have been exacerbated by the new LHC data that points to  $m_h \simeq 125 \text{ GeV}$ .

Next-to-leading order (NLO) constraints on SM vacuum stability based on two-loop renormalization group (RG) equations, one-loop threshold corrections at the electroweak scale (possibly improved with two-loop terms in the case of pure QCD corrections), and one-loop effective potential seem to indicate  $m_h \approx 125 - 126 \text{ GeV}$  saturates the minimum value that ensures a vanishing Higgs quartic coupling around the Planck scale ( $M_{\text{Pl}}$ ), see *e.g.* [12–22, 39]. However, the devil is in the details, a more recent NNLO analysis [23] yields a very restrictive condition of absolute stability up to the Planck scale

$$m_h > \left[ 129.4 + 1.4 \left( \frac{m_t/\text{GeV} - 173.1}{0.7} \right) - 0.5 \left( \frac{\alpha_s(m_Z) - 0.1184}{0.0007} \right) \pm 1.0_{\text{th}} \right] \text{ GeV}. \quad (1)$$

When combining in quadrature the theoretical uncertainty with experimental errors on the mass of the top ( $m_t$ ) and the strong coupling constant ( $\alpha_s$ ), one obtains  $m_h > 129 \pm 1.8 \text{ GeV}$ . The vacuum stability of the SM up to the Planck scale is excluded at  $2\sigma$  (98% C.L. one sided) for  $m_h < 126 \text{ GeV}$  [23]. Achieving the stability will necessarily impose constraints on physics beyond the SM.

Very recently we have put forward a (string based) Standard-like Model [24]. Motivated by the above, here we study the vacuum stability of its scalar sector. The layout of the paper is as follows. In Sec. II we briefly review the generalities of our model and we derive the RG equations. In Sec. III we present our results and conclusions.

## II. RG EVOLUTION EQUATIONS OF SM<sup>++</sup>

Very recently, we engineered the minimal extension of the SM that can be embedded into a Superstring Theory endowed with a high mass string scale,  $M_s \lesssim M_{\text{Pl}}$  [24]. The gauge extended sector,  $U(3)_B \times SU(2)_L \times U(1)_{I_R} \times U(1)_L$ , has two additional  $U(1)$  symmetries and thus we refer to our model as SM<sup>++</sup>. The origin of this model is founded on the D-brane structure of string compactifications, with all six extra dimensions  $\mathcal{O}(M_{\text{Pl}}^{-1})$  [25–27]. The low energy remnants of the D-brane structure are the gauge bosons and Weyl fermions living at the brane intersections of a particular 4-stack quiver configuration [28]. A schematic representation of the D-brane construct is shown in Fig. 1. The general properties of the chiral spectrum are summarized in Table I.

The resulting  $U(1)$  content gauges the baryon number  $B$  [with  $U(1)_B \subset U(3)_B$ ], the lepton number  $L$ , and a third additional abelian charge  $I_R$  which acts as the third isospin component of an  $SU(2)_R$ . Contact with gauge structures at TeV energies is achieved by a field rotation to couple diagonally to hypercharge  $Y_\mu$ . Two of the Euler angles are determined by this rotation and the third one is chosen so that one of the  $U(1)$  gauge bosons couples only to an anomaly free linear combination of  $I_R$  and  $B - L$ . Of the three original abelian couplings, the baryon number coupling  $g'_3$  is fixed to be  $\sqrt{1/6}$  of the QCD coupling  $g_3$  at the string scale. The orthogonal nature of the rotation imposes one additional constraint on the remaining couplings  $g'_1$  and  $g'_4$  [29]. Since one of the two extra gauge bosons is coupled to an anomalous current, its mass is  $\mathcal{O}(M_s)$ , as generated through some Stückelberg mechanism.<sup>1</sup> The other gauge boson is coupled to an anomaly free current and therefore (under certain topological conditions) it can remain massless and grow a TeV-scale mass through ordinary Higgs mechanisms [31].

Electroweak symmetry breaking is achieved through the standard Higgs doublet  $H$ . The spontaneous symmetry breaking of the extra non-anomalous  $U(1)$  is attained through an  $SU(2)$  singlet scalar field  $H''$ , which carries  $L$  and  $I_R$  numbers, and acquires a vacuum expectation value (VEV) at the TeV scale. With the charge assignments of Table I there are no dimension 4 operators involving  $H''$  that contribute to the Yukawa Lagrangian. This is very important since  $H''$  carries the quantum numbers of right-handed neutrino and its VEV breaks lepton number. However, this breaking can affect only higher-dimensional operators which are suppressed by the high string scale, and thus there is no phenomenological problem with experimental constraints for  $M_s$  higher than  $\sim 10^{14}$  GeV. Herein we remain agnostic with respect to supersymmetry (SUSY) breaking and the details of the low energy effective potential. However, we do subject the choice of quantum numbers for  $H''$  to the stringent holonomic constraints of the superpotential at the string scale. This forbids

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<sup>1</sup> A point worth noting at this juncture: SM can also be embedded in a 3-stack quiver comprising (only) one additional  $U(1)$  symmetry,  $U(3) \times SU(2) \times U(1)$  [30]. The extra gauge boson is anomalous and must grow a Stückelberg mass  $\sim M_s$ . In this D-brane model the running of the quartic Higgs coupling would reveal a vacuum instability around  $10^{10}$  GeV [23].

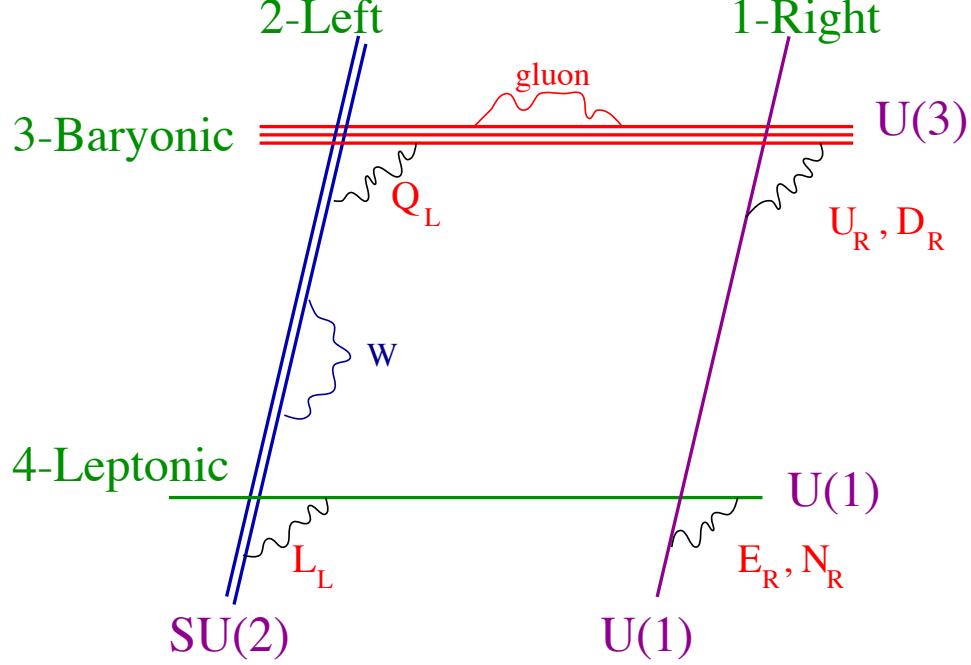


FIG. 1: Pictorial representation of the  $U(3)_B \times SU(2)_L \times U(1)_L \times U(1)_{I_R}$  D-brane model.

TABLE I: Chiral spectrum of  $SM^{++}$ .

Fields	Sector	Representation	$Q_B$	$Q_L$	$Q_{I_R}$	$Q_Y$
$U_R$	$3 \rightleftharpoons 1^*$	$(3, 1)$	1	0	1	$\frac{2}{3}$
$D_R$	$3 \rightleftharpoons 1$	$(3, 1)$	1	0	-1	$-\frac{1}{3}$
$L_L$	$4 \rightleftharpoons 2$	$(1, 2)$	0	1	0	$-\frac{1}{2}$
$E_R$	$4 \rightleftharpoons 1$	$(1, 1)$	0	1	-1	-1
$Q_L$	$3 \rightleftharpoons 2$	$(3, 2)$	1	0	0	$\frac{1}{6}$
$N_R$	$4 \rightleftharpoons 1^*$	$(1, 1)$	0	1	1	0
$H$	$2 \rightleftharpoons 1$	$(1, 2)$	0	0	1	$\frac{1}{2}$
$H''$	$4 \rightleftharpoons 1$	$(1, 1)$	0	-1	-1	0

the simultaneous presence of scalar fields and their complex conjugate. As an illustration, if the quantum numbers of  $H''$  are those of  $N_R^c$ , then higher dimensional operators such as  $\overline{N}_R N_R^c H''^2$ , which can potentially generate a Majorana mass, are absent. Because of holonomy this absence cannot be circumvented by including  $\overline{N}_R N_R^c H''^{*2}$ .

The scalar Lagrangian of  $SM^{++}$  is

$$\mathcal{L}_s = (\mathcal{D}^\mu H)^\dagger \mathcal{D}_\mu H + (\mathcal{D}^\mu H'')^\dagger \mathcal{D}_\mu H'' - V(H, H''), \quad (2)$$

where

$$V(H, H'') = \mu^2 |H|^2 + \mu'^2 |H''|^2 + \lambda_1 |H|^4 + \lambda_2 |H''|^4 + \lambda_3 |H|^2 |H''|^2 \quad (3)$$

is the potential and

$$\mathcal{D}_\mu = \partial_\mu - ig_3 T^a G_\mu^a - ig_3' Q_B C_\mu - ig_2 \tau^a W_\mu^a - ig_1' Q_{IR} B_\mu - ig_4' Q_L X_\mu \quad (4)$$

is (in a self-explanatory notation [24]) the covariant derivative in the field basis shown in Fig. 1. Next, we impose the positivity conditions [32]

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_1 \lambda_2 > \frac{1}{4} \lambda_3^2. \quad (5)$$

If the conditions (5) are satisfied, we can proceed to the minimisation of  $V(H, H'')$  as a function of constant VEVs for the two Higgs fields. Making use of gauge invariance, it is not restrictive to take

$$\langle H \rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h_1(x) \end{pmatrix} \quad \text{and} \quad \langle H'' \rangle \equiv \frac{1}{\sqrt{2}} (v'' + h_2(x)), \quad (6)$$

with  $v$  and  $v''$  real and non-negative. The physically most interesting solutions to the minimisation of (3) are obtained for  $v$  and  $v''$  both non-vanishing

$$v^2 = \frac{-\lambda_2 \mu^2 + \frac{1}{2} \lambda_3 \mu'^2}{\lambda_1 \lambda_2 - \frac{1}{4} \lambda_3^2} \quad \text{and} \quad v''^2 = \frac{-\lambda_1 \mu'^2 + \frac{1}{2} \lambda_3 \mu^2}{\lambda_1 \lambda_2 - \frac{1}{4} \lambda_3^2}. \quad (7)$$

To compute the scalar masses, we must expand the potential (3) around the minima (7). We denote by  $h$  and  $h''$  the scalar fields of definite masses,  $m_h$  and  $m_{h''}$  respectively, taking  $m_h \simeq 125$  GeV,  $v = 246$  GeV, and  $m_{h''} \gg m_h$ . After a bit of algebra, the explicit expressions for the scalar mass eigenvalues and eigenvectors are given by

$$m_h^2 = \lambda_1 v^2 + \lambda_2 v''^2 - \sqrt{(\lambda_1 v^2 - \lambda_2 v''^2)^2 + (\lambda_3 v v'')^2}, \quad (8)$$

$$m_{h''}^2 = \lambda_1 v^2 + \lambda_2 v''^2 + \sqrt{(\lambda_1 v^2 - \lambda_2 v''^2)^2 + (\lambda_3 v v'')^2}, \quad (9)$$

$$\begin{pmatrix} h \\ h'' \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad (10)$$

where  $\alpha \in [-\pi/2, \pi/2]$  also fulfils

$$\sin 2\alpha = \frac{\lambda_3 v v''}{\sqrt{(\lambda_1 v^2 - \lambda_2 v''^2)^2 + (\lambda_3 v v'')^2}}, \quad (11)$$

$$\cos 2\alpha = \frac{\lambda_1 v^2 - \lambda_2 v''^2}{\sqrt{(\lambda_1 v^2 - \lambda_2 v''^2)^2 + (\lambda_3 v v'')^2}}. \quad (12)$$

Now, it is convenient to invert (8), (9) and (11), to extract the parameters in the Lagrangian in terms of the physical quantities  $m_h$ ,  $m_{h''}$  and  $\sin 2\alpha$

$$\begin{aligned} \lambda_1 &= \frac{m_{h''}^2}{4v^2} (1 - \cos 2\alpha) + \frac{m_h^2}{4v^2} (1 + \cos 2\alpha), \\ \lambda_2 &= \frac{m_h^2}{4v''^2} (1 - \cos 2\alpha) + \frac{m_{h''}^2}{4v''^2} (1 + \cos 2\alpha), \\ \lambda_3 &= \sin 2\alpha \left( \frac{m_{h''}^2 - m_h^2}{2v v''} \right). \end{aligned} \quad (13)$$

As we noted above, the fields  $C_\mu, X_\mu, B_\mu$  are related to  $Y_\mu, Y'_\mu, Y''_\mu$  by an Euler rotation matrix [33],

$$\mathbb{R} = \begin{pmatrix} C_\theta C_\psi & -C_\phi S_\psi + S_\phi S_\theta C_\psi & S_\phi S_\psi + C_\phi S_\theta C_\psi \\ C_\theta S_\psi & C_\phi C_\psi + S_\phi S_\theta S_\psi & -S_\phi C_\psi + C_\phi S_\theta S_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{pmatrix}. \quad (14)$$

Hence, the covariant derivative for the  $U(1)$  fields in Eq. (4) can be rewritten in terms of  $Y_\mu, Y'_\mu$ , and  $Y''_\mu$  as follows

$$\begin{aligned} \mathcal{D}_\mu &= \partial_\mu - iY_\mu (-S_\theta g'_1 Q_{IR} + C_\theta S_\psi g'_4 Q_L + C_\theta C_\psi g'_3 Q_B) \\ &\quad - iY'_\mu [C_\theta S_\phi g'_1 Q_{IR} + (C_\phi C_\psi + S_\theta S_\phi S_\psi) g'_4 Q_L + (C_\psi S_\theta S_\phi - C_\phi S_\psi) g'_3 Q_B] \\ &\quad - iY''_\mu [C_\theta C_\phi g'_1 Q_{IR} + (-C_\psi S_\phi + C_\phi S_\theta S_\psi) g'_4 Q_L + (C_\phi C_\psi S_\theta + S_\phi S_\psi) g'_3 Q_B]. \end{aligned} \quad (15)$$

Now, by demanding that  $Y_\mu$  has the hypercharge

$$Q_Y = c_1 Q_{IR} + c_3 Q_B + c_4 Q_L \quad (16)$$

we fix the first column of the rotation matrix  $\mathbb{R}$

$$\begin{pmatrix} C_\mu \\ X_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} Y_\mu c_3 g_Y / g'_3 & \dots \\ Y_\mu c_4 g_Y / g'_4 & \dots \\ Y_\mu c_1 g_Y / g'_1 & \dots \end{pmatrix}, \quad (17)$$

and we determine the value of the two associated Euler angles

$$\theta = -\arcsin[c_1 g_Y / g'_1] \quad (18)$$

and

$$\psi = \arcsin[c_4 g_Y / (g'_4 C_\theta)], \quad (19)$$

with  $c_1 = 1/2$ ,  $c_3 = 1/6$ ,  $c_4 = -1/2$ ,  $B = Q_B/3$  and  $L = Q_L$ . The couplings  $g'_1$  and  $g'_4$  are related through the orthogonality condition,  $P(g_Y, g'_1, g'_3, g'_4) = 0$ , yielding

$$\left(\frac{c_4}{g'_4}\right)^2 = \frac{1}{g_Y^2} - \left(\frac{c_3}{g'_3}\right)^2 - \left(\frac{c_1}{g'_1}\right)^2, \quad (20)$$

with  $g'_3$  fixed by the relation of  $U(N)$  unification

$$g'_3(M_s) = \frac{1}{\sqrt{6}} g_3(M_s). \quad (21)$$

Next, by demanding that  $Y''_\mu$  couples to an anomalous free linear combination of  $I_R$  and  $B - L$  we determine the third Euler angle

$$\tan \phi = -S_\theta \frac{3 g'_3 C_\psi + g'_4 S_\psi}{3 g'_3 S_\psi - g'_4 C_\psi}. \quad (22)$$

The absence of abelian, mixed, and mixed gauge-gravitational anomalies is ensured utilizing the generalized Green-Schwarz mechanism, in which triangle anomalies are cancelled by Chern-Simons couplings. In the  $Y$ -basis we require the (mass)<sup>2</sup> matrix of the anomalous sector to be  $\text{diag}(0, M'^2, 0)$ . For the heavy field we take  $M' \sim M_s$  and therefore  $Y'_\mu \simeq Z'_\mu$

decouples from the low energy physics. The non-anomalous gauge boson,  $Y_\mu'' \simeq Z_\mu''$  grows a TeV-scale mass via  $H''$  [24].

Altogether, the covariant derivative of the low energy effective theory reads

$$\mathcal{D}_\mu = \partial_\mu - ig_3 T^a G_\mu^a - ig_2 \tau^a W_\mu^a - ig_Y Q_Y Y_\mu - ig_Y Q_Y Y_\mu'' - ig_{B-L} (B-L) Y_\mu'', \quad (23)$$

where

$$\begin{aligned} g_Y &= -\frac{1}{2} S_\theta g_1' = -\frac{1}{2} C_\theta S_\psi g_4' = \frac{3}{2} C_\theta C_\psi g_3' \\ g_Y &= 2 C_\theta C_\phi g_1' \\ g_{B-L} &= 3g_3'(C_\phi C_\psi S_\theta + S_\phi S_\psi) - C_\theta C_\phi g_1'. \end{aligned} \quad (24)$$

Finally, a straightforward calculation leads to the RG equations for the five parameters in the scalar potential [34]

$$\begin{aligned} \frac{d\mu^2}{dt} &= \frac{\mu^2}{16\pi^2} \left( 12\lambda_1 + 6Y_t^2 + 2\frac{\mu'^2}{\mu^2} \lambda_3 - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 - \frac{3}{2}g_Y^2 \right), \\ \frac{d\mu'^2}{dt} &= \frac{\mu'^2}{16\pi^2} \left( 8\lambda_2 + 4\frac{\mu^2}{\mu'^2} \lambda_3 - 24g_{B-L}^2 \right), \\ \frac{d\lambda_1}{dt} &= \frac{1}{16\pi^2} \left( 24\lambda_1^2 + \lambda_3^2 - 6Y_t^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_Y^4 + \frac{3}{4}g_2^2 g_Y^2 + \frac{3}{4}g_2^2 g_Y^2 + \frac{3}{4}g_Y^2 g_Y^2 + \frac{3}{8}g_Y^4 \right. \\ &\quad \left. + 12\lambda_1 Y_t^2 - 9\lambda_1 g_2^2 - 3\lambda_1 g_Y^2 - 3\lambda_1 g_Y^2 \right), \\ \frac{d\lambda_2}{dt} &= \frac{1}{8\pi^2} (10\lambda_2^2 + \lambda_3^2 + 48g_{B-L}^4 - 24\lambda_2 g_{B-L}^2), \\ \frac{d\lambda_3}{dt} &= \frac{\lambda_3}{8\pi^2} \left( 6\lambda_1 + 4\lambda_2 + 2\lambda_3 + 3Y_t^2 - \frac{9}{4}g_2^2 - \frac{3}{4}g_Y^2 - \frac{3}{4}g_Y^2 - 12g_{B-L}^2 \right) + \frac{3}{4\pi^2} g_Y^2 g_{B-L}^2, \end{aligned} \quad (25)$$

where  $t = \ln Q$  and  $Y_t$  is the top Yukawa coupling, with

$$\frac{dY_t}{dt} = \frac{Y_t}{16\pi^2} \left( \frac{9}{2}Y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_Y^2 - \frac{17}{12}g_Y^2 - \frac{2}{3}g_{B-L}^2 - \frac{5}{3}g_Y g_{B-L} \right) \quad (26)$$

and  $Y_t^{(0)} = \sqrt{2} m_t/v$ . The RG running of the gauge couplings follow the standard form

$$\begin{aligned} \frac{dg_3}{dt} &= \frac{g_3^3}{16\pi^2} \left[ -11 + \frac{4}{3}n_g \right] = -\frac{7}{16} \frac{g_3^3}{\pi^2}, \\ \frac{dg_2}{dt} &= \frac{g_2^3}{16\pi^2} \left[ -\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{6} \right] = -\frac{19}{96} \frac{g_2^3}{\pi^2}, \\ \frac{dg_Y}{dt} &= \frac{1}{16\pi^2} [A^{YY} g_Y^3], \\ \frac{dg_{B-L}}{dt} &= \frac{1}{16\pi^2} [A^{(B-L)(B-L)} g_{B-L}^3 + 2A^{(B-L)Y} g_{B-L}^2 g_Y + A^{YY} g_{B-L} g_Y^2], \\ \frac{dg_Y}{dt} &= \frac{1}{16\pi^2} [A^{YY} g_Y (g_Y^2 + 2g_Y^2) + 2A^{(B-L)Y} g_{B-L} (g_Y^2 + g_Y^2) + A^{(B-L)(B-L)} g_{B-L}^2 g_Y], \end{aligned} \quad (27)$$

where  $n_g = 3$  is the number of generations and

$$A^{ab} = A^{ba} = \frac{2}{3} \sum_f Q_{a,f} Q_{b,f} + \frac{1}{3} \sum_s Q_{a,s} Q_{b,s}, \quad (a, b = Y, B-L), \quad (28)$$

with  $f$  and  $s$  indicating contribution from fermion and scalar loops, respectively.

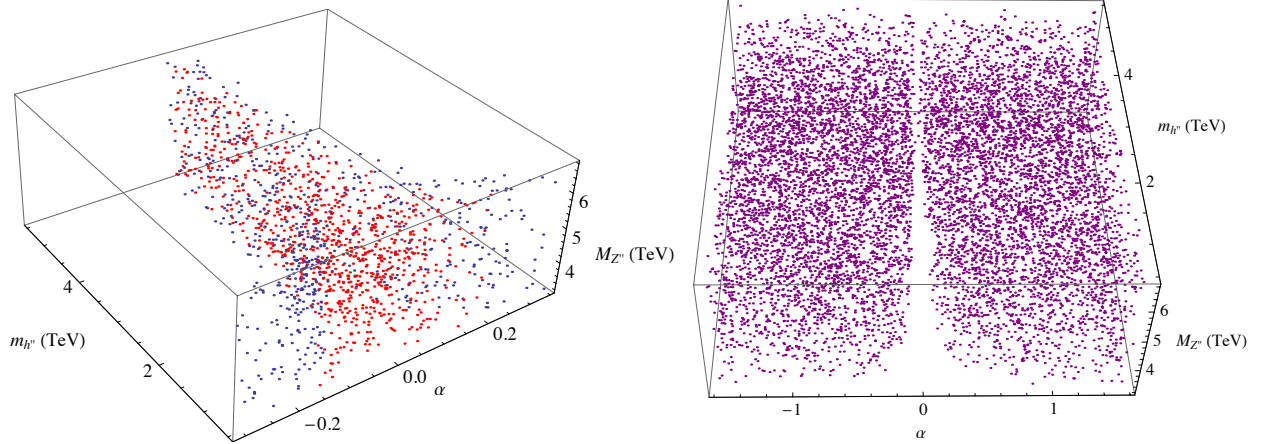


FIG. 2: An exhibition of the  $SM^{++}$  vacuum stability patterns in the  $m_{h''}$ ,  $\alpha$ ,  $M_{Z''}$  parameter space. The analysis is based on a scan of  $10^8$  trial random points with  $M_s = 10^{14}$  GeV. The points yielding a stable vacuum solution up to  $M_s$  are blue-printed, those leading to unstable vacuum solutions are red-printed, and points giving runaway solutions are purple-printed.

### III. RESULTS AND CONCLUSIONS

To ensure perturbativity of  $g'_4$  between the TeV scale and the string scale we find from (20) that  $g'_1 > 0.4845$ . We also take  $g'_1 \lesssim 1$  in order to ensure perturbativity at the string scale. Let us first study the region of the parameter space constrained by  $g'_1(M_s) \simeq 1$ . The string-scale values of the other abelian couplings are fixed by previous considerations (20) and (21). The Euler angles at  $M_s$  are also fixed by (18), (19), and (22). All the couplings and angles are therefore determined at all energies through RG running. As an illustration we set  $M_s = 10^{14}$  GeV; this leads to  $g'_3(M_s) = 0.231$ ,  $g'_4(M_s) = 0.232$ ,  $\psi(M_s) = -1.245$ ,  $\theta(M_s) = -0.217$ , and  $\phi(M_s) = -0.0006$ . Next, we define  $Q_{\min} = 4$  TeV and normalize  $t = \ln(Q/4 \text{ TeV})$  and  $t_{\max} = \ln(\Lambda/4 \text{ TeV})$ . Finally, we run the couplings and angles down to the TeV region:  $g'_1 = 0.406$ ,  $g'_3 = 0.196$ ,  $g'_4 = 0.218$ ,  $\theta = -0.466$ ,  $\psi = -1.215$ , and  $\phi = -0.0003$ .

There are *a priori* three free parameters to be fixed at the TeV-scale:  $(v'', \alpha, m_{h''})$ . The initial values of  $g_Y$ ,  $g_\gamma$  and  $g_{B-L}$  are then fixed by previous considerations (24). Actually, using the relation  $M_{Z''} = g'_1 C_\phi v''/C_\theta$  [24], we adopt  $(M_{Z''}, \alpha, m_{h''})$  as the free parameters of the model.<sup>2</sup> For  $M_s = 10^{14}$  GeV, we perform a scan of  $10^8$  trial random set of points,  $(M_{Z''}, \alpha, m_{h''})$ , and using (13) we obtain the initial conditions  $(\lambda_1^{(0)}, \lambda_2^{(0)}, \lambda_3^{(0)})$  to integrate (25). For each set of points, we verify that the positivity condition (5) is fulfilled all the way to  $\Lambda = M_s$ . The  $10^8$  trials are duplicated for  $M_s = 10^{16}$  and  $M_s = 10^{19}$  GeV. Our results are encapsulated in Figs. 2 to 9, and along with other aspects of this work are summarized in these concluding remarks:

- In Fig. 2 we show the results for the entire scan at  $M_s = 10^{14}$  GeV. The points yielding a stable vacuum solution up to  $M_s$  are blue-printed, those leading to unstable vacuum

<sup>2</sup> For  $M_s = 10^{14}$  GeV, the  $v'' \Rightarrow M_{Z''}$  relation implies that if  $7 \text{ TeV} < v'' < 15 \text{ TeV}$ , then  $3.16 \text{ TeV} < M_{Z''} < 6.78 \text{ TeV}$ . For a different  $M_s$  the range of  $M_{Z''}$  is altered because of changes in  $g'_1$ ,  $\theta$ , and  $\phi$ ; *e.g.* for  $M_s = 10^{19}$  GeV, the range becomes  $2.82 \text{ TeV} < M_{Z''} < 6.05 \text{ TeV}$ .



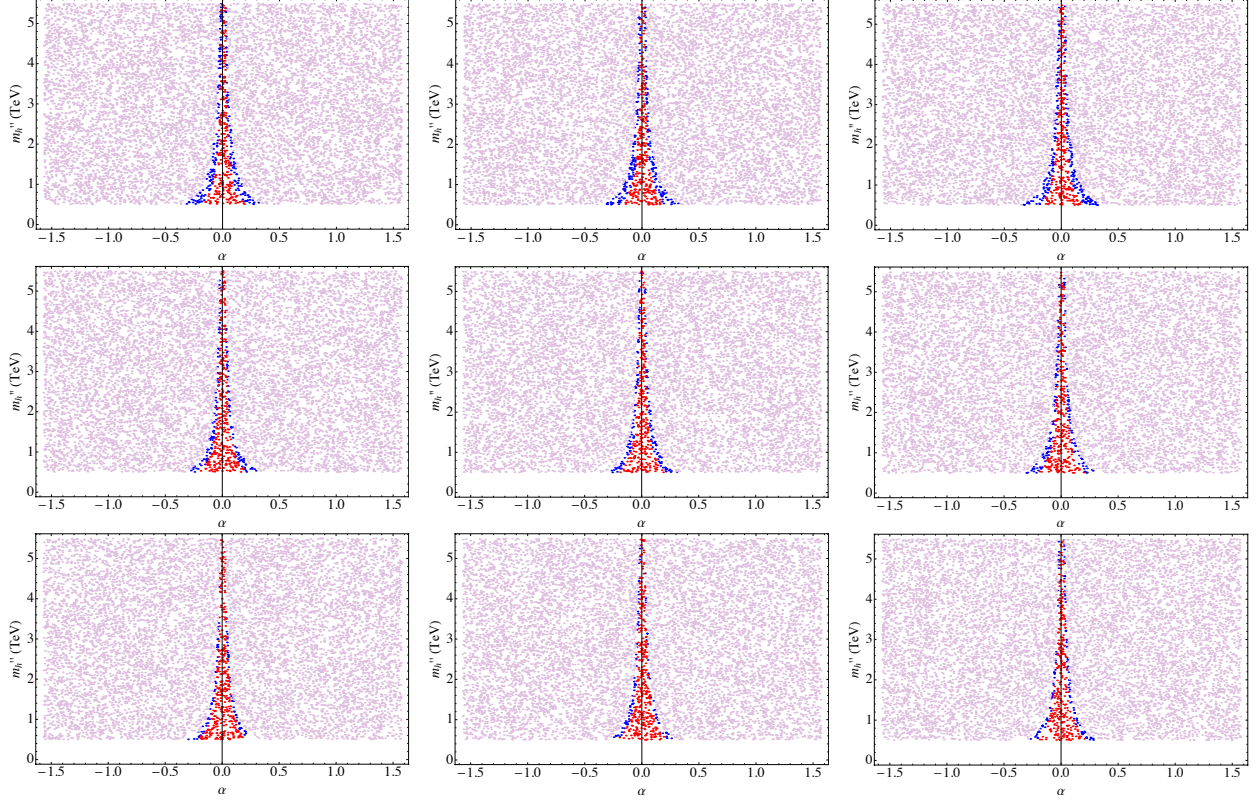


FIG. 3: An exhibition of the  $\text{SM}^{++}$  vacuum stability patterns in the  $m_{h''}$  vs  $\alpha$  plane, for fixed masses of  $Z''$ ;  $M_{Z''} = 3.5$  TeV (left pannels),  $M_{Z''} = 4.5$  TeV (middle pannels), and  $M_{Z''} = 6$  TeV (right pannels). The analysis is based on a scan of  $10^8$  trial random points with  $M_s = 10^{14}$  GeV (upper row),  $M_s = 10^{16}$  GeV (middle row), and  $M_s = 10^{19}$  GeV (lower row). The points yielding a stable vacuum solution up to  $M_s$  are blue-printed, those leading to unstable vacuum solutions are red-printed, and points giving runaway solutions (*i.e.*, those in which the Higgs doublet self-coupling blows up) are purple-printed.

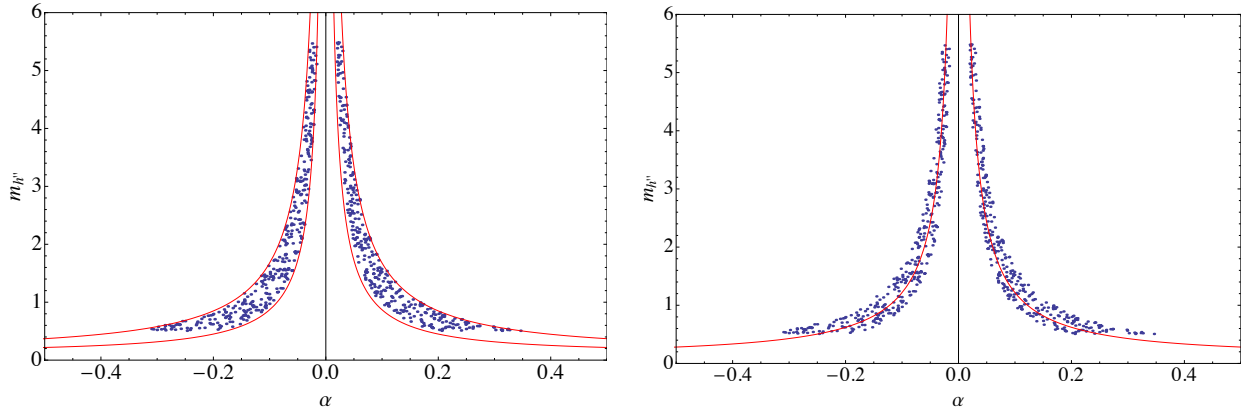


FIG. 4: Fits to the boundaries defining the region with stable vacuum solutions (left) and to the average value of the scatter points contained in that region (right).

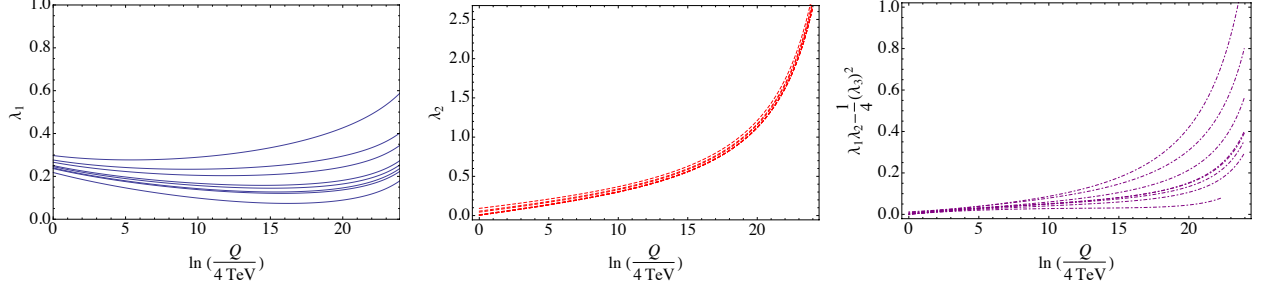


FIG. 5: Typical behavior of the running couplings  $\lambda_i(t)$ , for the average value of the initial condition,  $\langle \lambda_1^{(0)} \rangle = 0.25$  in the integration of (25).

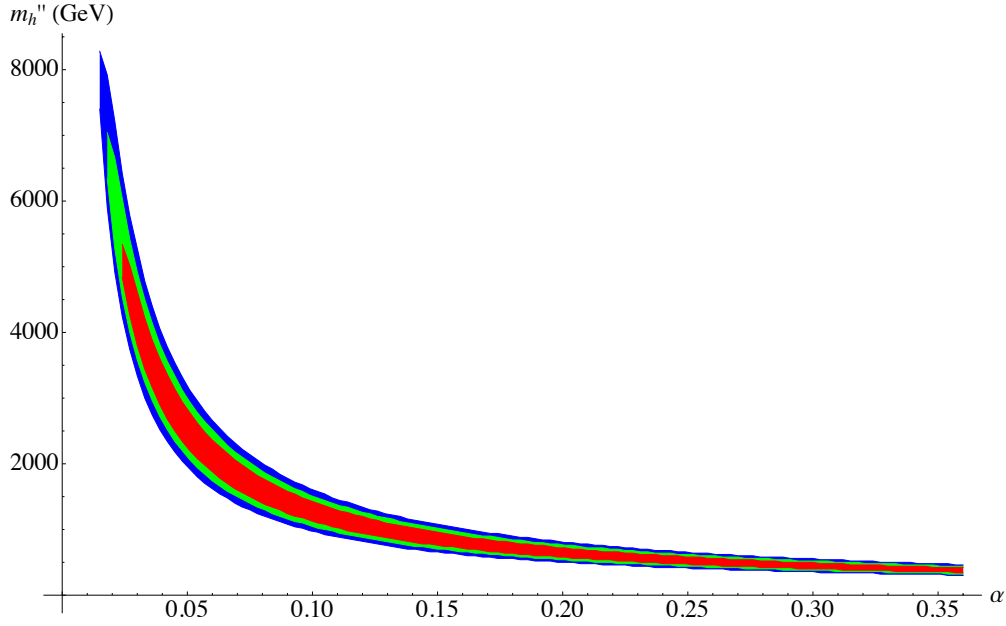


FIG. 6: The allowed  $\text{SM}^{++}$  parameter space in the  $m_{h''}$  vs  $\alpha$  plane under the vacuum stability constraint of Eq. (5), for the case  $M_{Z''} = 4.5$  TeV, with  $M_s = 10^{14}$  GeV (blue),  $M_s = 10^{16}$  GeV (green), and  $M_s = 10^{19}$  GeV (red). The perturbativity upper bound is defined by  $\lambda_i < 2\pi$ .

solutions are red-printed, and points giving runaway solutions are purple-printed. A stable vacuum solution is one in which the positivity condition (5) is fulfilled all the way to  $\Lambda = M_s$ . An unstable solution is one in which the stability conditions of the vacuum ( $\lambda_1 > 0$ ,  $\lambda_2 > 0$ ,  $\lambda_1\lambda_2 > \lambda_3^2/4$ ) are violated. A runaway solution is one in which the RG equations drive the Higgs doublet self-coupling non-perturbative. The perturbativity upper bound (sometimes referred to as ‘triviality’ bound) is given by  $\lambda_1 < 2\pi$  at any point in the RG evolution [20]. In the first row of Fig. 3 we display sections of constant  $M_{Z''}$  for the scan shown in Fig. 2. In the second and third rows of Fig. 3 we show three different sections of the  $m_{h''} - \alpha$  plane, corresponding to the same values of  $M_{Z''}$ , but for scans with different string scales:  $M_s = 10^{16}$  GeV (second row) and  $M_s = 10^{19}$  GeV (third row). The vacuum stability condition is driven by the behavior of  $\lambda_1$ , and actually is largely dominated by the initial condition  $\lambda_1^{(0)}$ . Indeed, if the extra gauge boson  $Z''$  gets its mass through a non-Higgs mechanism and the

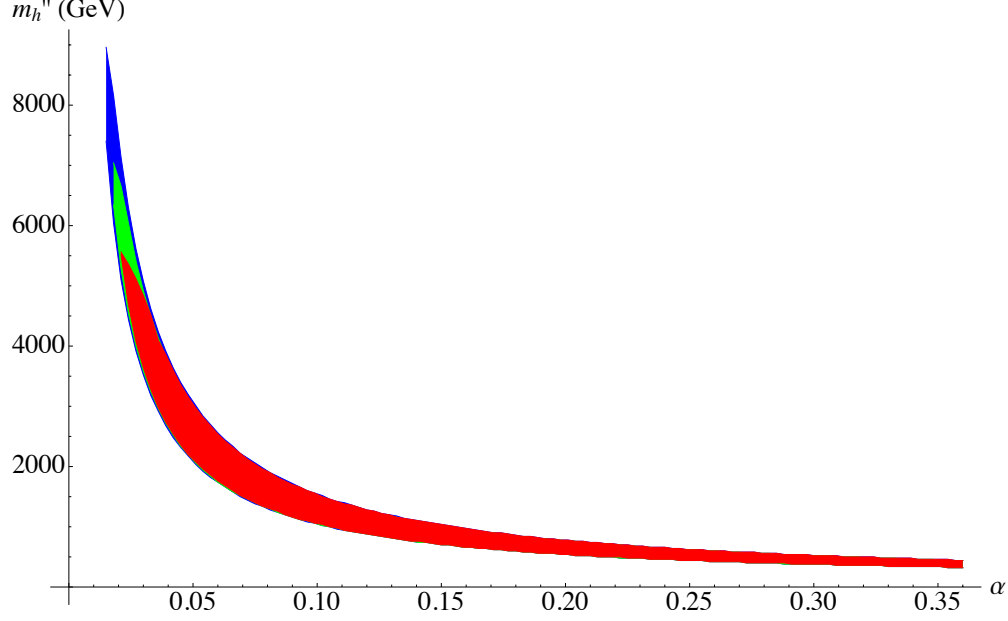


FIG. 7: Variation of  $\text{SM}^{++}$  vacuum stability regions with  $M_{Z''}$ . We have taken  $M_s = 10^{16}$  GeV,  $M_{Z''} = 3.5$  TeV (red),  $M_{Z''} = 4.5$  TeV (green), and  $M_{Z''} = 6.0$  TeV (blue). The perturbativity upper bound is defined by  $\lambda_i < 2\pi$ .

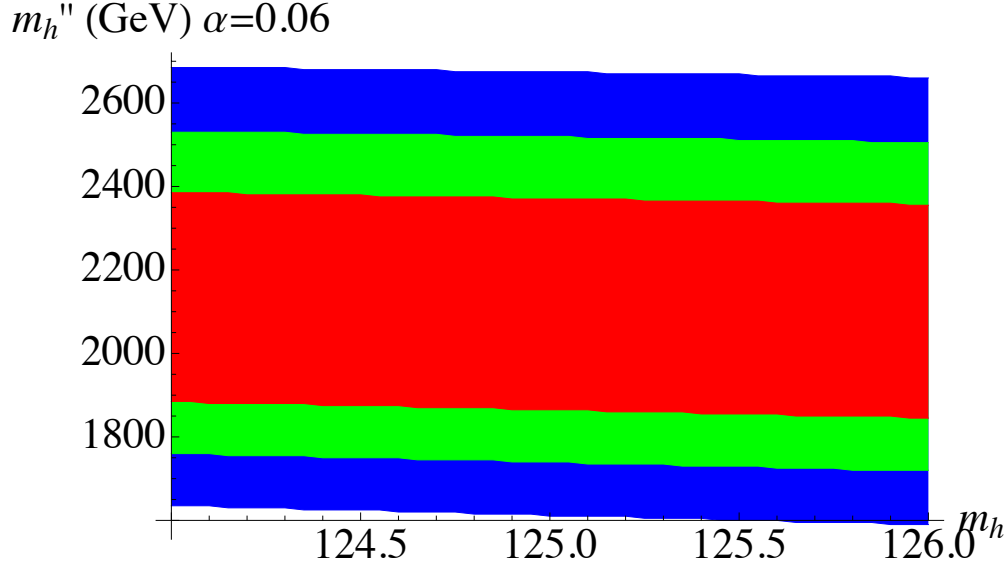


FIG. 8: Variation of  $\text{SM}^{++}$  vacuum stability regions with  $m_h$ . We have taken  $\alpha = 0.06$ ,  $M_{Z''} = 4.5$  TeV,  $M_s = 10^{14}$  GeV (blue),  $M_s = 10^{16}$  GeV (green),  $M_s = 10^{19}$  GeV (red). The perturbativity upper bound is defined by  $\lambda_i < 2\pi$ .

scalar potential (3) is that of SM (*i.e.*  $v'' = \lambda_2 = \lambda_3 = 0$ ), the RG evolution collapses to that of SM and there are no stable solutions.<sup>3</sup>

<sup>3</sup> Of course, even if  $v'' = \lambda_2 = \lambda_3 = 0$ , with an extra gauge boson the RG evolution of  $\lambda_1$  is not exactly

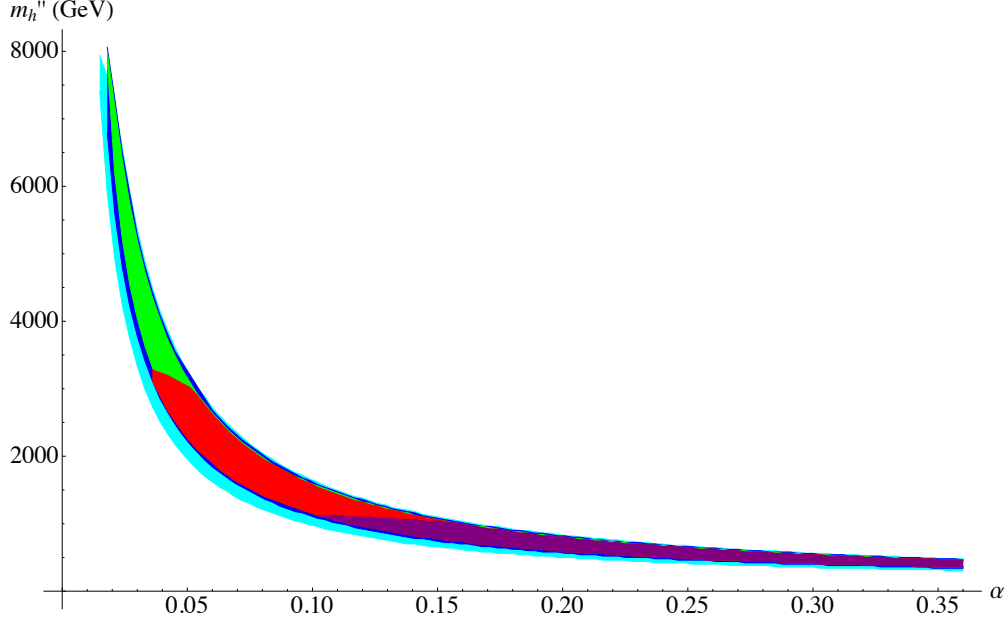


FIG. 9: Variation of SM<sup>++</sup> vacuum stability regions with  $g'_1(M_s)$ . The stable regions correspond to  $g'_1(M_s) = 1.0$  (cyan), 0.85 (blue), 0.6 (green), 0.485 (red), 0.4845 (purple). We have taken  $M_s = 10^{14}$  GeV,  $M_{Z''} = 4.5$  TeV,  $m_h = 125$  GeV. The perturbativity upper bound is defined by  $\lambda_i < 2\pi$ .

- To determine the range of initial conditions  $\lambda_1^{(0)}$  yielding stable vacuum solutions we fit the boundaries of the blue band in the scatter plot, with  $M_s = 10^{14}$  GeV. The results shown in the left panel of Fig. 4 lead to  $0.19 < \lambda_1^{(0)} < 0.36$ . The lower limit of  $\lambda_1^{(0)}$ , which defines the boundary between stable and unstable solutions, is close to the value required for vacuum stability of the SM potential, as shown in (1). Namely, substituting  $m_h = 130$  GeV and  $\alpha = 0$  in (13) we obtain  $\lambda_1^{(0)} = 0.14$ . The similarities between the minimum value of  $m_h$  that allows absolute stability up to the Planck scale within SM and the minimum value of  $m_h$  in the decoupling limit of (13) reinforces our previous statement concerning the strong dependence of the RG evolution with the initial condition  $\lambda_1^{(0)}$ . The allowed range of initial conditions for stable vacuum solutions depends on the value of the string scale; *e.g.* for  $M_s = 10^{19}$  GeV we obtain  $0.22 < \lambda_1^{(0)} < 0.30$ . We have also determined the average value of the initial condition  $\lambda_1^{(0)}$  through a fit to the blue points in the scattered plot. The results shown in the right panel of Fig. 4 lead to  $\langle \lambda_1^{(0)} \rangle = 0.25$ , independently of the value of  $M_s$ . The typical behavior of  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_1\lambda_2 - \lambda_3^2/4$ , for the average value of the initial condition  $\langle \lambda_1^{(0)} \rangle$ , is display in Fig. 5.
- To determine the sensitivity of the RG evolution with respect to the triviality bound we duplicate the analysis, but setting the perturbativity upper bound in all the couplings:  $\lambda_i < 2\pi$ . The contours display in Fig. 6 (for  $M_{Z''} = 4.5$  TeV) show there is almost no visible departure from the contours in Fig. 3.

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that of SM, see (25).

- In Fig. 7 we display the sensitivity of the RG evolution with  $M_{Z''}$ . For large values of  $\alpha$  there is no variation in the contour regions. For  $\alpha < 0.03$  there are some small variances. This small differences show the effect of the initial conditions  $\lambda_2^{(0)}$  and  $\lambda_3^{(0)}$  (both depending directly on  $M_{Z''}$ ) on the evolution of the system.
- We have verified that there is no significant variation of the  $\text{SM}^{++}$  vacuum stability regions within the  $m_h$  uncertainty. An example for  $\alpha = 0.06$  and  $M_{Z''} = 4.5$  TeV is given in Fig. 8.
- Finally, in Fig. 9 we display the variation of our results with  $g'_1(M_s)$ . It is clearly seen that for  $g'_1(M_s) \gtrsim 0.4845$  the dependence on  $g'_1$  seems to be fairly weak. However, if  $g'_1(M_s) < 0.4845$ , the  $g'_4$  coupling becomes non-perturbative and the stable region of parameters shrink dramatically. The stability of  $\text{SM}^{++}$  vacuum is then nearly independent of the  $Z''$  branching fractions [24].
- The low energy effective theory discussed in this paper requires a high level of fine tuning, which is satisfyingly resolved by applying the anthropic landscape of string theory [35–37]. Alternatively, the fine tuning can be circumvented with a more complete broken SUSY framework. Since in pure SUSY the vacuum is automatically stable, the stability analysis perforce involves the soft SUSY-breaking sector. Hence rather than simply searching for the Higgs self-coupling going negative in the ultraviolet, the stability analysis would involve finding the local and global minima of the effective potential in the multi-dimensional space of the soft-breaking sector [38]. However, the Higgs mass range favored by recent LHC data may be indicative of high-scale SUSY breaking [39]; perhaps near the high energy cutoff of the field theory, beyond which a string description becomes a necessity [40].
- In summary, we have shown that  $\text{SM}^{++}$  is a viable low energy effective theory, with a well-defined range of free parameters. It is of immediate interest to calculate the  $h$  couplings (after diagonalization) to  $W^+W^-$ ,  $b\bar{b}$ ,  $t\bar{t}$  and determine whether there is a region of the parameter space that would allow an enhancement in  $h \rightarrow \gamma\gamma$ , as hinted in LHC data [41].

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